

# Information for AP Calculus

Please fill out this form and return it to Ms. Midyette in Room 205 before leaving school for the summer. If you are unable to turn the form in, you may e-mail the information to: [AnneR.Midyette@vbschools.com](mailto:AnneR.Midyette@vbschools.com) by July 15.

Name \_\_\_\_\_

Address \_\_\_\_\_

Zip Code \_\_\_\_\_

Home Phone \_\_\_\_\_

Cell Phone \_\_\_\_\_

e-mail \_\_\_\_\_

Dates of any planned vacations or other commitments:

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Dear students,

Welcome to AP Calculus! You have chosen to take one of the most valuable mathematics courses that exists. Calculus incorporates all of the topics from your previous math courses and shows you how they can be used in the real world. Calculus is the basis for the study of change in every field. You will create a foundation on which to build in college or you might even fulfill your college math requirement completely.

**You need to complete the attached assignment and return it to me on or before our first class meeting in September. It is essential that you turn in the assignment on time or you may be removed from the course.**

Everything in the assignment comes from Math Analysis (or lower), but the questions may be presented in a different format than you are used to. Give your best attempt for each problem – I do not expect perfection. If you would like to work on the assignment with some of your classmates, I will be hosting a study session at Green Run in early August probably around 5:00. We will work on the assignment together and help each other get over the rough spots. This session is not mandatory, but I think you will learn a lot if you come. I will send you an invitation to the session in July, using the information that you provide on the cover sheet. Be sure you understand that it is OK to work together, but it is NOT acceptable to copy someone's work. If copying is your preferred method, you do not belong in an AP class.

Enjoy your summer – but don't forget about me!

*A. Madyette*

## REVIEW PROBLEMS

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1. The nautilus, a multi-chambered mollusk, the canal of the inner ear, and petals of certain flowers have the spiral shape shown in Figure 1.50.

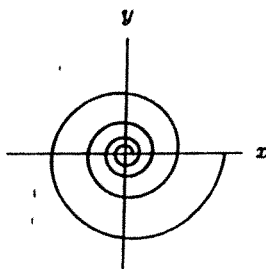


Figure 1.50

- (a) Is  $y$  a function of  $x$ ?  
(b) Is  $x$  a function of  $y$ ?  
(c) Is there any interval on the  $x$ -axis for which  $y$  is a function of  $x$ ?

2. Figure 1.51 represents the depth of the water at Montauk Point, New York for a given day in November. Using the graph determine the following:
- How many high tides took place on this day?
  - How many low tides took place on this day?
  - How much time elapsed in between high tides?

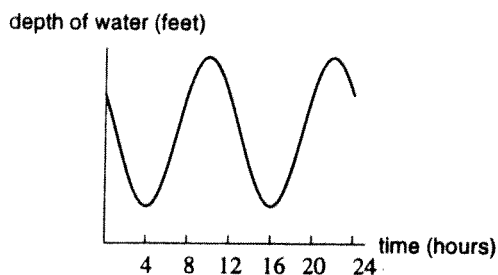


Figure 1.51

3. Figure 1.52 shows the average monthly temperature in Albany, New York, over a twelve-month period. (January is month 1.)
- Make a table showing average temperature as a function of the month of the year.
  - What is the warmest month in Albany?
  - Over what interval of months is the temperature increasing? Decreasing?

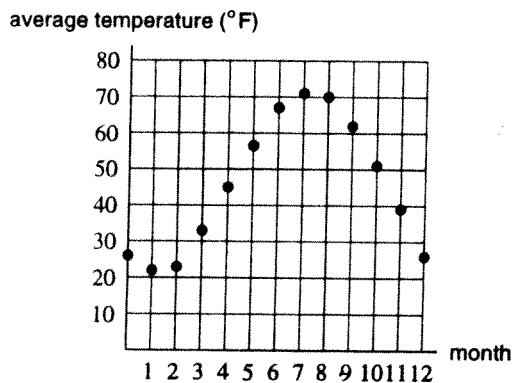


Figure 1.52

4. According to Jill Phillips, a home economist, the ideal size refrigerator for a family is 8 cubic feet for two people, plus 1 cubic foot for each additional member of the family for families with 3 through 10 members.<sup>12</sup>
- Construct a table showing the size of the ideal refrigerator as a function of the number of family members. From your table, what size refrigerator should a family of 5 own?
  - Plot a graph with refrigerator size on the vertical axis and the number of family members on the horizontal axis.
  - From your graph, estimate the ideal sized refrigerator for a family of 6.
  - Construct a formula which relates the size  $S$  to  $n$ , the number of family members.

<sup>12</sup>"Rules of Thumb" by T. Parker, Houghton Mifflin, Boston, 1983.

5. In 1947, Jesse Owens, the US gold medal track star of the 1930s and 1940s, ran a 100 yard race against a horse. The race, "staged" in Havana, Cuba, is filled with controversy; some say Owens received a head start, others claim the horse was drugged. Owens himself revealed some years later that the starting gun was placed next to the horse's ear causing the animal to rear and remain at the gate for a few seconds. Figure 1.53 depicts speeds measured against time for the race.
- (a) How fast were Owens and the horse going at the end of the race?  
 (b) When were the participants both traveling at the same speed?

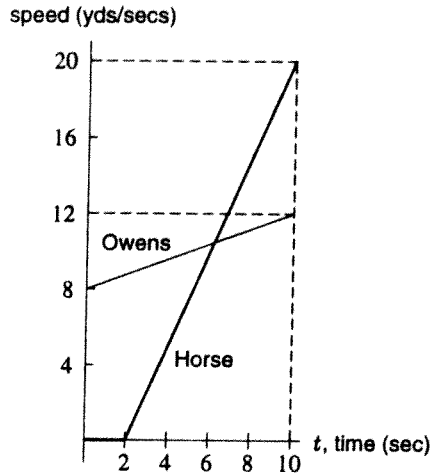


Figure 1.53

6. (a) Is the area,  $A$ , of a square a function of the length of one of its sides,  $s$ ?  
 (b) Is the area,  $A$ , of a rectangle a function of the length of one of its sides,  $s$ ?
7. Consider the graph in Figure 1.54. An open circle represents a point which is not included.
- (a) Is  $y$  a function of  $x$ ? Explain.  
 (b) Is  $x$  a function of  $y$ ? Explain.  
 (c) The domain of  $y = f(x)$  is  $0 \leq x < 4$ . What is the range of  $y = f(x)$ ?

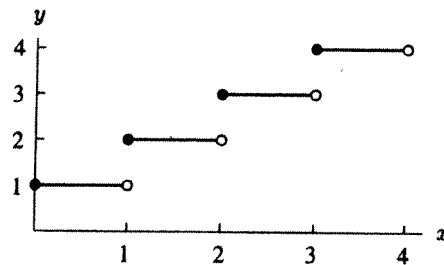


Figure 1.54

8. Is the graph of  $y = |x|$  concave up?

Omit

9. A person's blood sugar level at a particular time of the day is partially determined by the time of the most recent meal. After a meal, blood sugar level increases rapidly, then slowly comes back down to a normal level. Sketch a graph showing a person's blood sugar level as a function of time over the course of a day. Label the axes to indicate normal blood sugar level and the time of each meal.
10. Match each of the following descriptions with an appropriate graph and table of values.
- (a) The weight of your jumbo box of Fruity Flakes decreases by an equal amount every week.
  - (b) The machinery depreciated rapidly at first, but its value declined more slowly as time went on.
  - (c) In free fall, your distance from the ground decreases at an increasing rate.
  - (d) For a while it looked like the decline in profits was slowing down, but then they began declining ever more rapidly.

(A) 

<i>x</i>	0	1	2	3	4	5
<i>y</i>	400	384	336	256	144	0

(B) 

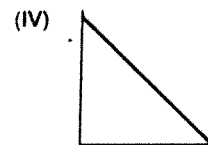
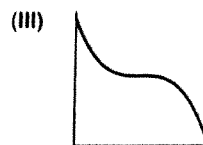
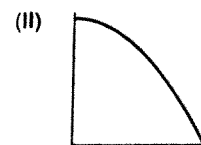
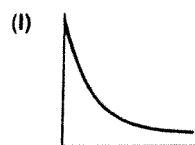
<i>x</i>	0	1	2	3	4	5
<i>y</i>	400	320	240	160	80	0

(C) 

<i>x</i>	0	1	2	3	4	5
<i>y</i>	400	184	98	63	49	43

(D) 

<i>x</i>	0	1	2	3	4	5
<i>y</i>	412	265	226	224	185	38



11. From December 1906 until July 1907, Bombay experienced a plague spread by rats. Table 1.40 shows the total number of deaths at the end of each month.<sup>13</sup>

**TABLE 1.40** *The total number of deaths at the end of each month during the Bombay plague*

Month	0	1	2	3	4	5	6	7
Deaths	4	68	300	1290	3851	7140	8690	8971

<sup>13</sup>Data extracted from "A contribution to the mathematical theory of epidemics" by W. O. Kermack and A. G. Mckendrick, Proc. Roy. Soc., 115A, 1927, pages 700-721

- (a) When was the number of deaths increasing? Decreasing?
- (b) For each successive month, construct a table showing the average rate of change in the number of deaths.
- (c) From the table you constructed in part (b) when will the graph of the number of deaths be concave up? Concave down? *(omit)*
- (d) When was the average rate of change of deaths the greatest? How is this related to part (c)? What does this mean in human terms as far as the spread of this epidemic is concerned?
- (e) Enter Table 1.40 on a graph and dash in a curve to help you see the trend in the data. From this graph identify where the curve is increasing, decreasing, ~~concave up, concave down~~. Compare your answers to those you found in parts (a) and (c). On the graph identify your answer to part (d), namely, the place when the average rate of change of deaths was greatest.
- (f) Ultimately 9100 people died due to this plague. Estimate at what time half this number was dead.
12. A price increases 5% due to inflation and is then reduced 10% for sale. Express the final price as a function of the original price,  $P$ .
13. The surface area of a cylindrical aluminum can is a measure of how much aluminum the can requires. If the can has radius  $r$  and height  $h$ , its surface area  $A$  and its volume  $V$  are given by the equations:

$$A = 2\pi r^2 + 2\pi r h \quad \text{and} \quad V = \pi r^2 h.$$

- (a) The volume,  $V$ , of a 12 oz cola can is  $355 \text{ cm}^3$ . A cola can is approximately cylindrical. Express its surface area  $A$  as a function of its radius  $r$ , where  $r$  is measured in centimeters. [Hint: First solve for  $h$  in terms of  $r$ .]
- (b) Sketch a graph of  $A = s(r)$ , the surface area of a cola can whose volume is  $355 \text{ cm}^3$ , for  $0 \leq r \leq 10$ . Label your axes.
- (c) What is the domain of  $s(r)$ ? Based on the sketch you made in (b), what, approximately, is the range of  $s(r)$ ?
- (d) The cola manufacturers wish to use the least amount of aluminum (in  $\text{cm}^2$ ) necessary to make a 12 oz cola can. Use your answer in (c) to determine the minimum amount of aluminum needed. State the values of  $r$  and  $h$  that would minimize the amount of aluminum used.
- (e) The radius of a real 12 oz cola can is about 3.25 cm. Show that real cola cans use more aluminum than necessary to hold 12 oz of cola. Why do you think cola cans are not made to these specifications?
14. Consider an 8-foot tall cylindrical water tank with a base of diameter 6 feet.
- (a) How much water can the tank hold?
- (b) How much water is in the tank if the water is 5 feet deep?
- (c) Write a formula for the volume of water as a function of its depth in the tank.
15. The purchase of batches of goods is a common practice in business. Goods are purchased periodically throughout the year in lots and put into storage. The stored goods are used as needed until the supply is exhausted or reaches a predetermined minimum, called the safety stock. Then a new order is placed.
- (a) Sketch a graph of time versus quantity in inventory without a safety stock.
- (b) On the same graph, draw the curve with a safety stock. Mark the interval between orders and the points when the shipment is received.

16. An incumbent politician running for reelection declared that the number of violent crimes is no longer rising and is presently under control. Does the graph shown in Figure 1.55 support this claim? Why or why not?

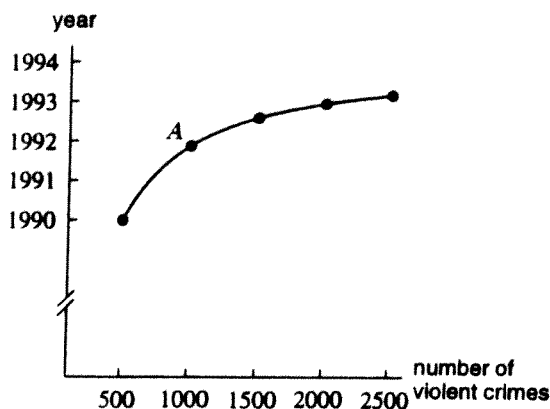


Figure 1.55

17. If  $g(x) = x\sqrt{x} + 100x$ , find  
 (a)  $g(100)$                       (b)  $g(4/25)$                       (c)  $g(1.21 \cdot 10^4)$
18. Let  $h(x) = x^2 + bx + c$ . Evaluate and simplify:  
 (a)  $h(1)$                                       (b)  $h(b + 1)$
19. For  $n$  a positive integer, the product  $1 \times 2 \times 3 \times \cdots \times (n - 1) \times n$  is called  $n$  factorial, and is written  $n!$ .  
 (a) Let  $p(n) = n!$  for  $n$  a positive integer. Evaluate  $p(n)$  for  $n = 1, 2, 3, \dots, 10$ . Compile your results in a table.  
 (b) Most calculators can evaluate  $n!$ . What is the largest value of  $n$  for which your calculator will compute  $n!$ ?
20. Let  $r(x)$  be the average number of pounds of trash recycled in a given week by those households producing a total of  $x$  pounds of trash in a week. For example,  $r(10)$  is the average amount of trash recycled per household among all households producing 10 pounds of trash per week.  
 (a) Give lower and upper bounds for  $r(q)$ , where  $q$  is some quantity of trash.  
 (b) A study indicates that, as a rule, when  $A > B$ ,  $r(A) > r(B)$ . The authors of the study conclude that households producing more trash are also more conscientious about recycling. Do you think the study supports this conclusion? If not, what additional evidence would be needed?
21. Find the domain and range for each of the following functions:  
 (a)  $m(x) = 9 - x$                       (b)  $n(x) = 9 - x^4$                       (c)  $q(x) = \sqrt{x^2 - 9}$
22. The cruising speed  $V$  of birds at sea-level (in meters/sec) is determined<sup>14</sup> by the mass  $M$  of the bird (in grams), and the surface area  $S$  of the wings exposed to the air (in square meters). It is given by

$$V = 0.164\sqrt{\frac{M}{S}}.$$

<sup>14</sup>"The Simple Science of Flight" by H. Tennekes, MIT Press, Cambridge, 1996.

#22 is  
extra  
credit

- (a) The mass of a partridge is half the mass of a hawk. Their wing surface areas are typically 0.043 and 0.166 square meters, respectively. Which bird has the faster cruising speed? The cruising speed of the partridge is 15.6 meters/sec. What are the masses of the partridge and the hawk?
- (b) The wing surface area of a Canadian goose is typically 12 times that of an American robin, whereas the mass of the goose is 70 times that of the robin. Which bird has the faster cruising speed? The mass and cruising speed of the American robin are typically 80 grams and 9.5 meters/sec, respectively. What are the wing surface areas of the Canadian goose and the American robin?
- (c) Using a graphing calculator or computer to plot  $V$  against different masses  $M$ , for birds with the same surface area of 0.01 square meters—swallows, martins, swifts, and so on. How would you describe the graph in words? What happens to the cruising speed as the mass increases?
- (d) Use a graphing calculator or computer to plot  $V$  against different wing surface areas  $S$ , for birds with the same mass 784 grams—falcons, hawks, and so on. How would you describe the graph in words? What happens to the cruising speed as the wing surface area increases?
- (e) When a bird dives it draws in its wings. What happens to its cruising speed? Is this realistic?
23. In Example 3 we saw that the period,  $p$ , of a pendulum is proportional to the square of its length,  $l$ .
- (a) The pendulum in a grandfather clock is 3 feet long and has a period of 1.924 seconds. Find the constant of proportionality, and write  $p$  as a function of  $l$ ,  $p = f(l)$ .
- (b) Graph  $p = f(l)$  for  $0 < l < 250$ .
- (c) The period of Foucault's pendulum, built in 1851 in the Pantheon in Paris, was 15.59 seconds. Find the length of the pendulum, and write your answer using function notation. Locate the corresponding point on your graph.
24. A certain subway train serves 19 stations, numbered 1 through 19. Although passengers can board the train at any of its 19 stops, most of its passengers board at the first station. The train has a capacity of 700 people. Define  $p(n)$  to be the number of passengers on a given day who board the train at the first station and exit at station number  $n$  or lower.
- (a) State the domain and range of  $p(n)$ .
- (b) Interpret the expression  $p(1)$ . Describe a scenario where  $p(1) > 0$ .
- (c) Is  $p(n)$  increasing, decreasing, neither, or can't we tell? Explain.
25. While white-water rafting, the guide in a kayak accompanies the raft. At points where the river is narrow, the kayak moves ahead of the raft, then waits for the raft to catch up. Where the river is wide, the kayak stays alongside the raft. The raft moves faster in regions where the river is narrow than in regions where the river is wide. Suppose that a kayak and a raft travel down the Reventazón River in Costa Rica. The river is narrow for the first half-mile, then wide for the next three quarters of a mile, then narrow for another two miles, and lastly wide for a mile.
- (a) Sketch the graph of the raft's distance from its starting point as a function of time. Label the narrow and wide regions of the river on your graph.
- (b) On the same graph as in part (a), sketch the position of the kayak as a function of time.

26. Academics have suggested that loss of worker productivity can result from sleep deprivation. An article in the Sunday, September 26, 1993, *New York Times* quotes David Poltrack, the senior vice president for planning and research at CBS, as saying that seven million Americans are staying up an hour later than usual to watch talk show host David Letterman. The article goes on to quote Timothy Monk, a professor at the University of Pittsburgh School of Medicine, as saying "... my hunch is that the effect [on productivity due to sleep deprivation among this group] would be in the area of a 10 percent decrement." The article next quotes Robert Solow, a Nobel prize-winning professor of economics at MIT, who suggests the following procedure to estimate the impact that this loss in productivity will have on the US economy – an impact he dubbed "the Letterman loss." First, Solow says, we find the percentage of the work force who watch the program. Next, we determine this group's contribution to the gross domestic product (G.D.P.). Then we reduce the group's contribution by 10% to account for the loss in productivity due to sleep deprivation. The amount of this reduction is "the Letterman loss."
- The article estimated that the G.D.P. is \$6.325 trillion, and that 7 million Americans watch the show. Assume that the nation's work force is 118 million people and that 75% of David Letterman's audience belongs to this group. What percentage of the work force is in Dave's audience?
  - What percent of the G.D.P. does David Letterman's audience contribute? How much money do they contribute?
  - How big is "the Letterman Loss"?
27. Table 1.41 shows the population of Ireland <sup>15</sup> at various times between 1780 and 1910, where 0 corresponds to 1780.

**TABLE 1.41** *The population of Ireland from 1780 to 1910, where 0 corresponds to 1780*

Year	0	20	40	60	70	90	110	130
Population (millions)	4.0	5.2	6.7	8.3	6.9	5.4	4.7	4.4

- When was the population increasing? Decreasing?
- For each successive time interval, construct a table showing the average rate of change of the population.
- From the table you constructed in part (b), when is the graph of the population concave up? Concave down?
- When was the average rate of change of the population the greatest? The least? How is this related to part (c)? What does this mean in human terms?
- Graph the data in Table 1.41 and join the points by a curve to help you see the trend in the data. From this graph identify where the curve is increasing, decreasing, concave up and concave down. Compare your answers to those you got in parts (a) and (c). Identify the region you found in part (d).
- Something catastrophic happened in Ireland between 1780 and 1910. When? Do you know what happened in Ireland at that time to cause this catastrophe?

<sup>15</sup> Adapted from "Mathematical Models in the Social, Management and Life Sciences" by D. N. Burghes and A. D. Wood, Ellis Horwood, 1980, page 104.